RAN-2003000204040057

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S. Y. B. Sc. (Sem. - IV) Examination April - 2023

Elective Generic : 4002

Mathematical Modeling

Time: 2 Hours] **[Total Marks: 50**

સૂચના : / Instructions

(1)

નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fill up strictly the details of ☛ signs on your answer book

Name of the Examination:

☛ **S. Y. B. Sc. (Sem. - IV)**

Name of the Subject :

☛ **Elective Generic : 4002 Mathematical Modeling**

Subject Code No.: **2003000204040057**

Seat No.:

Student's Signature

(2) All Questions are compulsory.

(3) First question carry 8 marks and other carries 14 marks.

(4) Figures to the right indicate marks of the corresponding question.

(5) Follow usual notations.

(6) Use scientific non programmable calculator.

Q. 1. Answer the following: (Any Four) **8**

1. Let $x(t) = x(0)e^{rt}$. If $x(t) = 1$ then find $x(0)$.

2. State Fick's law of diffusion.

3. If $\frac{d^2x}{dt^2} = a - 2bx$ then find the condition at which $\frac{d^2x}{dt^2} < 0$ and $\frac{d^2x}{dt^2} > 0$.

4. Differentiate $f(x, y, a) = 0$; a is parameter partially with respect to x.

5. Find the Orthogonal trajectories of the family of curves $y = -x$.

6. State the equation of the family of equiangular spirals.

Q. 2. **Answer the following: (Any Two)** **14**

(a) Obtain population growth model and solve it. Find the time at which the population will becomes half of its present size.

(b) Let the rate of growth of Bacteria is proportional to population of Bacteria at time t. If initially there are 1000 Bacteria and after 2 hours, the number of Bacteria is 2000 then find the population size of Bacteria after 5 hours and four hours.

(c) State Newton's law of cooling and solve it. A body where temperature T is initially $300°C$ is placed in a large block of ice. Find its temperature at the end of 2 and 3 minutes?

Q. 3. **Answer the following: (Any Two)** **14**

(a) Derive Logistic law of population growth model and solve it.

(b) Obtain the model of spread of Infectious diseases. Prove that after very long time all persons will be effected.

(c) Let $\frac{dN}{dt} = KN(R - N)$. If $K = 0.007$, $R = 1000$, $N(0) = 50$ then find $N(10)$ and $N(t) = 500$.

Q. 4. **Answer the following: (Any Two)** **14**

(a) Find the Orthogonal trajectories of the family of curves $x^2 + y^2 - 2ax = 0$.

(b) (i) Obtain the Orthogonal trajectories of the family of curves $r = a(1 + cos\theta)$.

(ii) Find curves for which the projection of the normal on the x – axis is of constant length.

(c) (i) Find curves for which tangent at a point is always perpendicular to the line joining the point to the origin.

(ii) Obtain the Orthogonal trajectories of the family of curves $y = mx$; m is parameter.

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